Elliptic Curves as Collapse-Stable Structures: A QCG Interpretation of Rank, Height, and Analytic Signature

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Abstract

Elliptic curves occupy a central position in modern mathematics, exhibiting a deep interplay between algebraic, arithmetic, and analytic structure. In this note, we propose a structural reinterpretation of elliptic curves within the framework of Quantum Collapse Geometry (QCG). Rather than introducing new arithmetic results, we provide a unifying conceptual scaffold in which elliptic curves are understood as collapse-stable invariant sectors selected under algebraic constraint. Within this perspective, the Mordell–Weil group is interpreted as a closure-generated persistence algebra, the rank as a measure of surviving generative dimensionality, the canonical height as a persistence functional, and the L-function as an analytic spectral encoding of structure. This interpretation situates elliptic curves within a broader generative framework in which stable structure arises through selection under constraint.

1 Introduction

Elliptic curves arise at the intersection of algebraic geometry, number theory, and analysis. Given by cubic equations of the form

$$E : y^2 = x^3 + ax + b, \tag{1}$$

they exhibit a rich structure encoded in invariants, group laws, and analytic functions. Despite their classical formulation, several aspects of their behavior—particularly the distribution of ranks and the structure of rational points—suggest an underlying pattern of constraint and selection.

Quantum Collapse Geometry (QCG) provides a general framework in which stable structure emerges from the selection of admissible configurations under constraint. In this note, we interpret elliptic curves within this framework, not as a replacement for standard theory, but as a structural reinterpretation that unifies their algebraic, arithmetic, and analytic features.

2 Layer I: Admissibility and Nondegeneracy

An elliptic curve is defined by the condition that its discriminant is nonzero:

$$\Delta = -16(4a^3 + 27b^2) \neq 0. \tag{2}$$

This condition excludes singular (degenerate) curves and ensures smoothness. From a QCG perspective, we interpret this as a selection condition acting on the configuration space

$$\Sigma = \{(a, b)\}. \tag{3}$$

Define a conceptual collapse operator

$$\Phi : \Sigma \to \Sigma_{\text{admissible}}, \tag{4}$$

which removes degenerate configurations ($\Delta = 0$).

Interpretation. The discriminant defines a boundary between admissible and non-admissible configurations. Elliptic curves correspond to collapse-stable invariant sectors selected under this constraint.

3 Layer II: Closure and the Mordell–Weil Structure

The rational points of an elliptic curve form a finitely generated abelian group:

$$E(\mathbb{Q}) \cong E(\mathbb{Q})_{\text{tors}} \oplus \mathbb{Z}^r, \tag{5}$$

where r is the rank.

From a QCG perspective, the group law defines a closure operation on admissible configurations.

Mapping:

Classical Structure	QCG Interpretation
Torsion subgroup	Closed (periodic) invariant loops
Free generators	Persistent generative directions
Rank r	Persistence dimensionality

Interpretation. The Mordell–Weil group is the closure algebra of admissible configurations. The rank measures the number of independent directions along which structure can continue to generate without collapsing into periodic closure.

4 Layer III: Persistence Geometry via Height and Regulator

The canonical height function

$$\hat{h}(P) \tag{6}$$

measures the arithmetic growth of a point under iteration.

QCG Interpretation.

$$\hat{h}(P) \sim \text{persistence of a generative direction.} \tag{7}$$

Low height corresponds to near-collapse behavior, while high height indicates strongly persistent structure.

For a curve of rank r, the regulator is defined as

$$\text{Reg}(E) = \det(\langle P_i, P_j \rangle), \tag{8}$$

where $\langle \cdot, \cdot \rangle$ is the height pairing.

QCG Interpretation.

$$\text{Reg}(E) \sim \text{coherent volume of the persistent sector.} \tag{9}$$

Interpretation. Height defines a persistence functional on arithmetic trajectories, while the regulator measures the effective volume of the surviving generative structure.

5 Layer IV: Analytic Signature and the L-Function

The L-function of an elliptic curve encodes deep arithmetic information. The Birch–Swinnerton-Dyer conjecture relates its behavior at $s = 1$ to the rank:

$$\mathrm{ord}_{s=1} L(E, s) = r. \tag{10}$$

QCG Mapping:

Classical Structure	QCG Interpretation
$L(E, s)$	Spectral encoding of structure
Zero at $s = 1$	Degeneracy / critical point
Order of zero	Persistence dimensionality

Interpretation. The order of vanishing of the L-function acts as a spectral signature of the dimensionality of the persistent arithmetic sector.

6 Rank Distribution as a Selection Law

Empirically, high-rank elliptic curves are rare. Within this framework, this reflects a constraint-driven selection effect.

Each additional independent generator must satisfy:

- nondegeneracy (admissibility),
- rational consistency,
- closure compatibility,
- global coherence constraints.

Interpretation. Rank behaves as a survival statistic: higher-dimensional generative structure is increasingly suppressed under constraint, leading to a natural bias toward low-rank curves.

7 Conclusion

Within the QCG framework, elliptic curves can be interpreted as collapse-stable invariant sectors selected under algebraic constraint. Their structure decomposes naturally into layers:

- Admissibility: nondegeneracy selects stable sectors,
- Closure: the Mordell–Weil group encodes persistent generativity,
- Geometry: height and regulator measure persistence structure,
- Analysis: the L-function encodes a spectral signature of persistence.

Synthesis. The rank of an elliptic curve can be interpreted as the dimensionality of arithmetic generativity that survives under global constraint. Height and regulator define the geometry of this persistence, while the L-function provides its analytic encoding.

This perspective does not replace classical theory, but situates elliptic curves within a broader generative framework in which stable structure arises through selection under constraint.

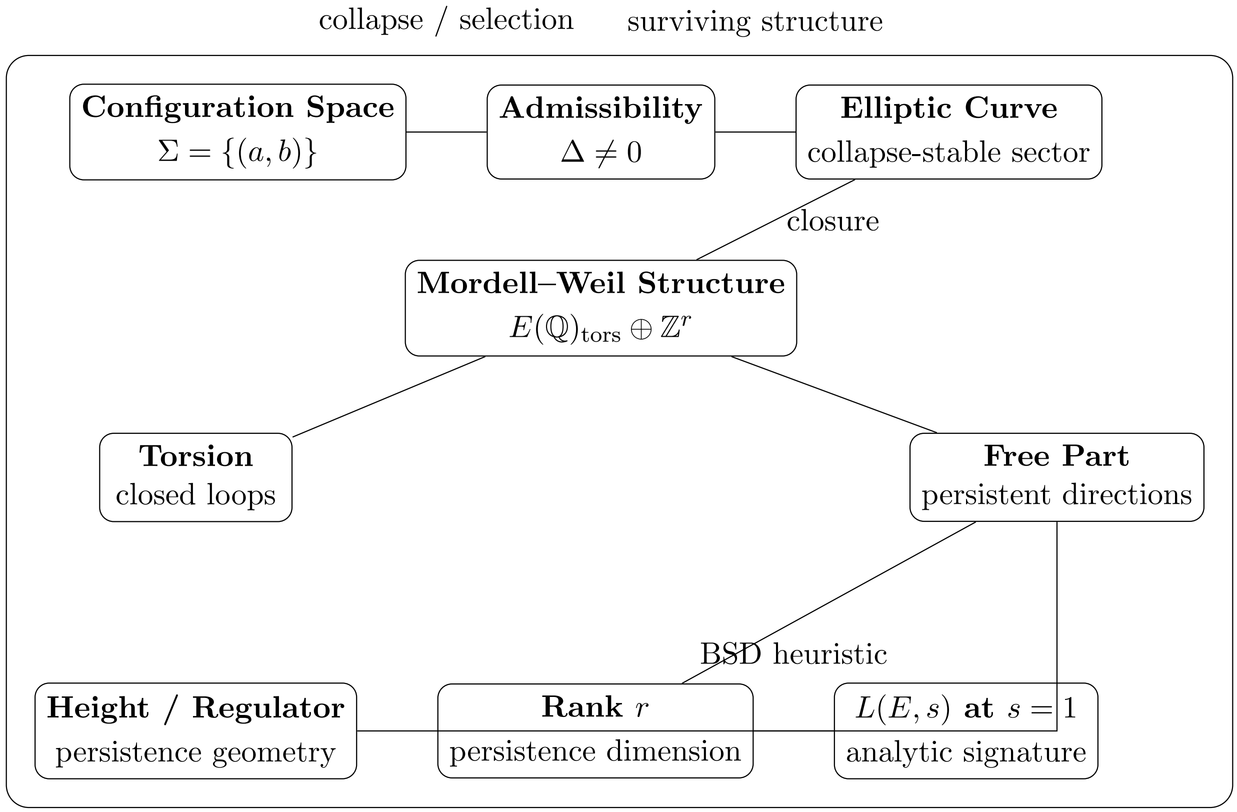

Figure 1: Conceptual mapping of elliptic-curve structure in QCG terms. A configuration space of coefficients is restricted by the nondegeneracy condition $\Delta \neq 0$, selecting an admissible elliptic sector. The Mordell–Weil decomposition is then interpreted as a closure structure, with torsion corresponding to closed invariant loops and the free part corresponding to persistent generative directions. The rank measures the dimensionality of this surviving sector, height and regulator encode its persistence geometry, and the behavior of $L(E, s)$ at $s = 1$ provides its analytic signature.

Future Directions

- Formal mapping between discriminant structure and collapse operators,
- Interpretation of height pairing as a dynamical or energy-like functional,
- Statistical modeling of rank distribution via constraint selection,
- Extension to higher-dimensional abelian varieties and motivic structures.